International Journal of Mathematics
and Computer Applications Research (IJMCAR)
ISSN(P): 2249-6955; ISSN(E): 2249-8060
Vol. 4, Issue 2, Apr 2014, 29-36

AN OPTIMAL POLICY INVENTORY MODEL WITH LOT SIZE DEPENDANT HOLDING COST AND SHORTAGES

P. PARVATHI[1] & K. PRASANNA LAKSHMI[2]

[1]Head & Associate Professor, Quiad-E-Millath Government College for Women (Autonomous), Chennai, Tamil Nadu, India

[2]Assistant Professor, Department of Mathematics, Ethiraj College for Women (Autonomous), Chennai, Tamil Nadu, India

ABSTRACT

Harris's inventory model treated the associated costs as constant and did not depend on any quantity. This paper explores the classical model with lot size dependant carrying/holding cost with allowable shortage. Carrying cost is directly proportional to lot size in steps. Numerical examples elucidate this model by applying an algorithm.

KEYWORDS: Inventory Model, Lot Size, Carrying, Holding Cost, Shortage Cost

INTRODUCTION

Inventory is very important for any business. Almost every business must store goods to ensure smooth and efficient running of its operation. Finding an appropriate system for a given inventory condition requires an expansive over view of inventory literature with thorough understanding of each model. The business scenario may vary with buyer and vendor. Sometimes the vendor who manufactures the product may fall shortage due to breakdown of machines to supply to the buyer which in turn is a shortage of buyer.

The Harris-Wilson formula for determining the optimum lot size is the pioneering model in the inventory systems. **Bingham. P [4]** had found the economic order quantities with step function ordering cost production and inventory management. The original EOQ model considers 3 type of costs: cost of the product, holding or carrying cost and ordering or set-up cost. Shortages refer to inability to meet the demand at the required time schedule as preferred by the customer.

Most of the research in the inventory modelling area is based on the relaxation of the original assumptions of Wilson's formula. Several researchers deal with modifications of the cost structures G**upta [10]** discussed an inventory model with lot size dependant ordering cost. The works done by **Gordon [9], Emergy [7]**, **Arrow K. Etal; [1]** in this field are worth mentioning.

The proposed model is developed from Harris Classical inventory model where the associated costs were taken as constant and did not depend on any quantity. This paper explores the classical model with lot size dependant carrying cost with allowable shortages. Wilson's formula determines optimum lot size, the quantity in which an item of inventory should be purchased or produced for stock. In this paper though the inventory system involves one type of item of product, **Chern, Chung-Mei Ho** [5] developed a multiproduct joint ordering model with dependent set-up cost.

For the wholesalers and manufacturers an item for which sales are of unit size and at a constant rate is the exception. When sales vary in size and take place irregularly at the time and the size of the sales are uncertain. This paper considers a situation where the carrying cost depends on the lot size with allowable shortages.

There are many real life situations where carrying cost depends on the lot size and various extensions of non-constant carrying cost can be seen in **Beraneck [3]**. To suit the real life situation we allow shortages for both vendor and buyer. **Golabi K.[8]** had found the optimal inventory policies when ordering policies are random and **Kaoi N. and Osaki S. [6]** had also developed the optimal ordering policies when order cost depend on time.

As the lot-size increases the carrying cost also increases. When items are received they are to be checked and inspected etc. These costs would generally increase as the size of the lot increase and it would cost the organisation certain fixed cost such as wages per hour or per shift for both loading and unloading the items. The labour charges would increase as the run-size increases. The carrying cost includes all the above costs and hence we consider lots size dependant carrying cost. For a concave set-up large scale system, **Bensoussan A. and Proth J.M [2]** had done inventory planning in a deterministic environment.

In this paper an inventory model has been developed where the carrying cost depends on the lot-size. As the lot size increases stepwise the carrying cost also increases. This paper is developed from the paper “An inventory model with lot-size dependant carrying /holding cost”, by **Karabi Dutta Choudhary, Summit Saha and Mantu Das [11]**, where the shortages are not allowed. In any trade shortage of items occurs frequently so concept of shortage of items is considered. In this paper by allowing the shortages it is shown that the optimal lot size and the total cost are minimized compared with the base paper.

NOTATIONS AND ASSUMPTIONS

For the proposed model we consider the following notations and assumptions:

D = Total/Annual demand

W= Shortage quantity;

q_j = Range of order quantity;

Q_j = Feasible order quantity

W'= Shortage quantity corresponding to Q_j

C_1 = The carrying cost for a unit for one year

C_2 = The shortage cost per unit

C_3 = The setup cost per order

$T_C(Q_j, W)$ = Total cost in Rupees

ASSUMPTIONS

- The inventory system involves one type of item or product.
- The demand is known and constant.
- Lead time is constant and known (i.e.) replenishment is instantaneous.
- Shortages are allowed and corresponding shortage cost is involved.
- C1< C2< C3

MODEL DEVOLOPMENT ANALYSIS

From Wilson's formula the order quantity and shortage quantity are given by

$$Q_j = \sqrt{\frac{2DC_3}{C_1} + W^2 + \frac{C_2W^2}{C_1}} \quad (1)$$

$$W = \sqrt{\frac{2C_3 \quad DC_1}{C_2(C_2+C_1)}} \quad (2)$$

The total cost is given by

$$\mathbf{T_c (Q_j, W)} = \frac{(D\,C_3)}{q_j} + \frac{C_1 \;(q_j + W)^2}{2q_j} + \frac{C_2 \; W^2}{2q_j} \quad (3)$$

We take Q_j

If $q_{j-1} \leq Q_j \leq q_j$

Where j= 1, 2, 3,...m, $q_0 = 0$ and $q_m = \infty$

For carrying cost C_1, EOQ is given by:

$$Q_j = \sqrt{\frac{2DC_3}{C_1} + W'^2 + \frac{C_2W'^2}{C_1}} \quad (4)$$

If Q_j does not lie within the interval $[q_{j-1}, q_j]$ i.e. Q_j is not order feasible, then the optimal lot size will be determined by

q_{j-1} if $Q_j \leq q_{j-1}$ (4.1)

q_j if $Q_j \geq q_j$ (4.2)

With the known values of Q_j, $T_C (Q_j, W)$ can be calculated from the equation (3).

If Q_j is feasible then $T_C (Q_j, W)$ will be the optimal lot-size. Otherwise the value of Q_j thus obtained by equations (4.1) and (4.2). $T_C (Q_j, W)$ are calculated by equation (3). Thus among all the calculated values of $T_C (Q_j, W)$, the smallest value will be the optimal cost i.e. $Tc (Q_{opt}, W)$ and the respective Q_j will be the optimal lot size i.e. Q_{opt}

Now the total cost is minimum

$$\frac{\partial T_c \;(Q_j,W)}{\partial Q_j} = 0; \frac{\partial T_c \;(Q_j,W)}{\partial B} = 0;$$

$$\left(\frac{\partial^2 T_c}{\partial Q_j^2}\right)\left(\frac{\partial^2 T_c}{\partial B^2}\right) - \frac{\partial^2 T_c}{\partial Q_j . \partial_B} > 0$$

$$\frac{\partial^2 T_c \;(Q_j,W)}{\partial Q_j^2} > 0; \frac{\partial^2 T_c \;(Q_j,B)}{\partial w^2} > 0$$

From equation (3)

$$\frac{\partial T_c(Q_j,W)}{\partial Q_j} = \frac{-DC_3}{Q_j} + \frac{C_1}{2} - \frac{C_1 W^2}{2Q_j{}^2} - \frac{C_2 W^2}{2Q_j^2}$$

$$\frac{\partial T_c(Q_j,W)}{\partial Q_j} = 0$$

$=>$ (i e) $\frac{-DC_3}{Q_j} + \frac{C_1}{2} - \frac{C_1 W^2}{2{Q_j}^2} - \frac{C_2 W^2}{2Q_j^2} = 0$

$$=> \frac{-2DC_3 + Q_j^2 C_1 - C_1 W^2 + C_2 W^2}{2Q_j^2} = 0$$

$$=> \quad -2DC_3 + Q_j^2 C_1 - C_1 W^2 + C_2 W^2 = 0$$

$$=> \quad Q_j^2 C_1 = 2DC_3 + C_1 W^2 + C_2 W^2$$

$$=> Q_j^2 = \frac{2DC_3}{C_1} + \frac{C_1 W^2}{C_1} + \frac{C_2 W^2}{C_1}$$

$$\boldsymbol{Q_j} = \sqrt{\frac{2DC_3}{C_1} + W^2 + \frac{C_2 W^2}{C_1}}$$

Now which is same as (1)

Now find $\frac{\partial T_c}{\partial W}$ and equate it to zero:

$$\frac{\partial T_c(Q_j, W)}{\partial W} = 0 + \frac{2C_1 (Q_j - W)(-1)}{2Q_j} + \frac{2WC^2}{2Q_j}$$

$$= -\frac{C_1 (Q_j - W)}{Q_j} + \frac{WC_2}{Q_j}$$

$$= \frac{-C_1 Q_j}{Q_j} + \frac{C_1 W}{Q_j} + \frac{WC_2}{Q_j}$$

$$= \quad -C_1 + \frac{C_1 W}{Q_j} + \frac{WC_2}{Q_j}$$

Now $\frac{\partial T_c}{\partial (Q_j, W} = 0$

$$-C_1 + \frac{C_1 W}{Q_j} + \frac{WC_2}{Q_j} = 0$$

$$\frac{W}{Q_j} (C_1 + C_2) = C_1$$

Therefore W $(C_1 + C_2) = C_1 Q_j$

Therefore

$$\mathbf{W}' = \frac{C_1 Q_j}{C_1 + C_2} \tag{5}$$

We have

$$\frac{\partial T_c(Q_j, W)}{\partial Q_j} = -\frac{DC_3}{Q_j^2} + \frac{C_1}{2} - \frac{C_1 W^2}{2{Q_j}^2} - \frac{C_2 W^2}{2\partial Q_j^2}$$

$$\frac{\partial^2 T_c(Q_j, W)}{{Q_j}^{\ 2}} = \frac{2DC_3}{Q_j^3} + 0 + \frac{2C_1 W^2}{2{Q_j}^3} + \frac{2C_2 W^2}{2\partial Q_j^3}$$

$$= \frac{2DC_3 + C_1 W^2 + C_2 W^2}{Q_j^3} > 0$$

$$\frac{\partial T_C(Q_j,W)}{\partial W} = -C_1 + \frac{C_1 W}{Q_j} + \frac{C_2 W}{Q_j}$$

$$\frac{\partial^2 T_C(Q_j,W)}{\partial W^2} = 0 + \frac{C_1}{Q_j} + \frac{C_2}{Q_j}$$

$$= \frac{C_1 + C_2}{Q_j} > 0$$

We know that

$$\frac{\partial T_C(Q_j,W)}{\partial Q_j} = -\frac{DC_3}{Q_j^2} + \frac{C_1}{2} - \frac{C_1 W^2}{2Q_j^2} - \frac{C_2 W^2}{2Q_j^2}$$

$$\frac{\partial^2 T_C(Q_j,W)}{\partial Q_j \partial W} = \frac{2C_1 W}{2Q_j^2} - \frac{2C_2 W}{2Q_j^2}$$

$$= -\frac{C_1 W}{Q_j^2} - \frac{C_2 W}{Q_j^2}$$

$$= -\frac{W(C_1 + C_2)}{Q_j^2}$$

Also

$$\left(\frac{\partial^2 T_C}{\partial {Q_j}^2}\right) \cdot \left(\frac{\partial^2 T_C}{\partial W}\right) - \frac{\partial^2 T_C}{\partial Q_j \partial W} > 0$$

Hence the condition is verified.

ALGORITHM

- Set j = 1
- Input the initial value for Qj. say ( )
- Put in equation 5 and find '
- Put in equation 1 find Q1.
- Put in equation 5 find
- Repeat the above steps, until and are consistent.
- Corresponding to this, find feasible value according to the (feasible) conditions.
- Find corresponding to feasible
- Put feasible and in total cost and hence find the optimal cost.

Example 1

D= 1800 unit

C3 = Rs. 350/-

Carrying costs and shortage costs are given in the following table:

Table 1

J	$Q_j = \sqrt{\frac{2DC_3}{C_1} + W'^2 + \frac{C_2W'^2}{C_1}}$	RANGE q_j	FEASIBLE Q_j	C_1	C_2	C_3	W'	$T_C(Q_j, W')$
1.	107.76	5-30	30	110	160	350	12	21,978
2.	104.66	31-55	60	130	180	350	23	13,529.82
3.	105.88	56-81	81	150	200	350	35	11,249.38
4.	100.21	82-107	107	180	220	350	37	11,416.73

For the lot-size Q_{opt}= 81 units approximately and T_C (Q_{opt}) =Rs.11, 249/-

Example: 2

D = 1000 units

C_3 = Rs.350/-

Table 2

J	$Q_j = \sqrt{\frac{2DC_3}{C_1} + W'^2 + \frac{C_2W'^2}{C_1}}$	RANGE q_j	FEASIBLE Q_j	C_1	C_2	C_3	W'	$T_C(Q_j, W')$
1.	81.96	5-30	30	110	160	350	12	12,644.67
2.	81.52	31-55	55	130	180	350	23	7764.37
3.	86.75	56-81	81	150	200	350	35	7793.60
4.	82.19	82-107	107	180	220	350	37	8799.91

Carrying costs and shortage costs are given in the following table:

For the lot-size Q_{opt}= 55 units approximately and TC (Q_{opt}) =Rs.7764.31/

Example: 3

D = 1000 units

C_3 = Rs.200/

Carrying costs and shortage costs are given in the following table:

Table 3

J	$Q_j = \sqrt{\frac{2DC_3}{C_1} + W'^2 + \frac{C_2W'^2}{C_1}}$	RANGE q_j	FEASIBLE Q_j	C_1	C_2	C_3	W'	$T_C(Q_j, W')$
1.	63.16	5-30	30	110	160	350	12	7644.67
2.	65.87	31-55	55	130	180	350	23	5712.18
3.	74.30	56-81	81	150	200	200	35	5940.75
4.	72.54	82-107	82	180	220	200	37	6068.96

For the lot-size Q_{opt}= 55 units approximately and TC (Q_{opt}) =Rs.5712.18

CONCLUSIONS

In this paper the classical Harris-Wilson model has been extended with carrying cost depending on lot-size and with allowable shortages. It is observed and calculated that ,if the value of Q_j lying within the interval i.e. order feasible, it will give the optimal costs. And if no such value of Q_j can be obtained which is order feasible, then we can make order feasible by equations (4.1) and (4.2) and out of all the calculated values of the total cost, the minimum total cost will give the optimal order quantity. Also it may be observed that the optimal cost depends on the demand required as well as the set-up cost per order. In day today life shortage of items is natural in many business scenario. Cost savings is achieved in this model by allowing shortage.

REFERENCES

1. Arrow. K., Harrisk. T. and Marschak. J (1951), Optimal Inventory Policy, Econometrica, 19,250-272.
2. Bensoussan. A, and Proth. J. M. (1984), Inventory planning in a deterministic environment; concave set-up, Large scale- system, 6, 177-184
3. Beranek. W. (1967), Financial Implecation of lot size inventory models, Management Science, 20 1081-1091.
4. Bingham. P. (1986), Economic order quantities for systems with step function ordering costs, production and inventory management, 27, 119-127.
5. Chern, Chung-Mei Ho, (1974), a multi-product joint ordering model with dependent setup cost, Management Science, 20, 1081-1091. Kaio. N. and Osaki. S, (1978), optimal ordering policies when order costs depend on time, RAIRO, 12, 93-99
6. Emery. J. C. (1954), some Theoretical Aspect of Inventory control, Master’s Thesis in Industrial Management M.I.T
7. Golabi. K. (1985), optimal inventory policies when ordering prices are random, Operations Research, 33, 575-588.
8. Gorden. M. J, and Taylor W. J. (1956), the condition for lot-size production, J. Of American Statistical Association vol 51, No. 276, 627-636
9. Gupta.K.Omprakash (1994), An Inventory model with lot-size dependent ordering cost, Production Planning and Control, 6, 585-587.
10. Karabi Dutta Choudhary, Sumit Saha and Mantu Das (2011), An Inventory Model with lot Size Dependent carrying/holding cost, Assam University Journal of Science and Technology, 133-136, 2011.